**Elementary Row Operations:**

1. Interchange two rows
2. Multiply a row by a nonzero real number
3. Replace a row by its sum with a multiple of another row

**Over-determined System**: More equations than unknowns. In R2 each equation represents a line, usually inconsistent.

**Underdetermined System:** A system with fewer equations than unknowns. Usually consistent with infinitely many solutions.

**Inconsistent System:** A system with no solutions.

**Consistent System:** A system with one or infinitely many solutions.

**Reduced Row Echelon Form:**

1. If the first nonzero entry in each nonzero row is 1.
2. If row k does not consists entirely of zeros, the number of leading zero entries in row k+1 is greater than the number of leading zero entries in row k.
3. If there are rows whose entries are all zero, they are below the rows having nonzero entries.

**Matrix-vector multiplication** : If A is an mxn matrix and x is a vector in Rn then

Ax = x1a1 + x2a2 + … + xnan

**Consistency Theorem:** A linear system Ax=b is consistent if and only if b can be written as a linear combination of the column vectors of A.

**Matrix-Matrix Multiplication:**

1. A + B = B + A
2. (A + B) + C = A + (B + C)
3. (AB)C = A(BC)
4. A(B + C) = AB + AC
5. (A + B)C = AC + BC
6. (αβ)A = α(βA)
7. α(AB) = (αA)B = A(αB)
8. (α + β)A = αA + βB
9. α(A + B) = αA + αB

Generally AB ≠BA (not commutative)

**Identity Matrix:** The nxn iidentitty matrix is the matrix I = aij where

Aij = 1 if i = j

= 0 if i ≠ j

**Invertible/Nonsingular**: An nxn matrix A is invertible if there exists a matrix B such that AB = BA = I. the matrix B is said to be a multiplicative inverse of A.

If A and B are nonsingular nxn matrices, then AB is also nonsingular and (AB)-1 = B-1A-1

PROOF

(B-1A-1)AB = B-1(A-1A)B = B-1B = I

(AB)(B-1A-1) = A(BB-1)A-1 = AA=1 = I

**Singular Matrix:** An nxn matrix that does not have a multiplicative inverse.

**Transpose Matrix**: Transpose of an mxn matrix A is the nxm matrix B defined by: bji = aij for

j=1,…,n and i=1,…,m.

1. (AT)T = A
2. (αA)T = αAT
3. (A+B)T = AT + BT
4. (AB)T = BTAT

**Symmetric Matrix**: An nxn matrix where AT = A

**Elementary Matrix:** The result of performing exactly one elementary row operation on an identity matrix.

1. Obtained by interchanging two rows of I.
   1. Multiplying A on the left by E (EA) interchanges the rows of A.
   2. Multiplying A on the right by E (AE) interchanges the columns of A.
2. Obtained by multiplying a row of I by a nonzero constant.
   1. Multiplication on the left by E (EA) performs the elementary row operation of multiplying the row.
   2. Multiplication on the right by E (AE) performs the elementary column operation of multiplying the third column by 3.
3. Obtained by adding a multiple of one row to another row.
   1. Multiplication on the left by E adds the rows.
   2. Multiplication on the right by E adds to the column.

**Inverse**: If E is an elementary matrix, then E is nonsingular and E-1 is an elementary matrix of the same type.

**Row Equivalent**: A matrix B is row equivalent to A if there exists a finite sequence E1,E2,…,Ek of elementary matrices such that B = EkEk-1…E1A

-If A is row equivalent to B, then B is row equivalent to A.

-If A is row equivalent to B and B is row equivalent to C, then A is row equivalent to C.

**Equivalent Conditions for Nonsingularity**: Let A be an nxn matrix. The following are equivalent:

-A is nonsingular.

-Ax = 0 has only the trivial solution 0.

-A is row equivalent to I.

Corollary: The system of n linear equations in n unknowns Ax=b has a unique solution if and only if A is nonsingular.

**Minors and Cofactors**: Let A = (aij) be an nxn matrix and let Mij denote the (n-1)x(n-1) matrix obtained from A by deleting the row and column containing aij. The determinant of Mij is called the minor or aij.

We define the cofactor Aij of aij by Aij = (-1)i+jdet(Mij)

**Determinant**: the determinant of an nxn matrix A, denoted det(A) is a scalar associated with the matrix A that is defined intuitively as follows:

det(A) = a11 if n=1

= a11A11+a12A12+…+a1nA1n if n>1

Where Aij = (-1)1+jdet(M1j) j=1,…,n

are the cofactors associated with the entries in the first row of A.

det(AT) = det(A):

If A has a row or column consisting entirely of zero, then det(A)=0

The determinant of a triangular matrix equals the product of the diagonal elements

If A has two identical rows or columns, then det(A) = 0.

**An nxn matrix A is singular iff**: det(A) = 0

**Properties of Determinants:**

1. Interchanging 2 rows = change the sign of the determinant
2. If we multiply a row by a scalar α, the determinant is also multiplied by α
3. If we add a multiple of another row, the determinant doesn’t change.
4. If A has one row (or columns) that is a multiple of another, the determinant is zero.
5. If A and B are nxn matrices then det(Ab) = det(A)det(B).
6. If A is a nonsingular matrix then det(A-1) = 1/(det(A))